CONTRAIL PHYSICS

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September 17 2000

A preliminary model has been developed to estimate the length of time that is required for a contrail to dissipate. It is assumed within this discussion that the contrail is composed of water vapor (per historical definition). The model developed agrees extremely well with the historical behavior and observation of contrails. The model is not intended to encompass all variables that may be in effect, but it does model reasonably well the expected behavior of water at flight altitudes. Any errors will be corrected if and as they are brought to my attention. It will be noted that this model is not a function of relative humidity, as no basis from thermodynamics has yet been established for it's inclusion. Any model based upon the premise of "mixing" as the primary mechanism for dissipation requires quantification to receive consideration. Cloud formation and the introduction of aerosol particles to assist in their formation is an entirely different discussion which is to be examined separately. The conclusions that result from the study of this model are several:

1. Contrails composed of water vapor routinely dissipate, as the physics and chemistry of this model will demonstrate. As a separate and distinct set of events, clouds may form if temperature, relative humidity, and aerosol conditions are favorable to their development. If "contrails" by appearance transform into "clouds", it can be concluded that the material of composition is not water vapor.

2.The conditions under consideration show that the ice crystals within a contrail can warm to the melting point and subsequently melt with the heat provided by solar radiation.

3. As demonstrated both by historical observation and this model, the time expected for contrail dissipation is relatively short, e.g., 2 minutes or less. This assumes the contrail is composed essentially of water vapor, per the classic definition (condensed trail).

4. The rate of contrail dissipation is highly dependent upon the the size of the ice crystal particles and the amount of solar radiation. Dependence upon relative humidity is not evident. 'Cloud' formation from aircraft, should it occur, is dependent primarily upon the temperature, the relative humidity, and the type and size of aerosol particles(nuclei) that are introduced.

The basic form of the contrail dissipation model, based upon the chemistry, mathematics and physics of thermodynamics is as follows:

time for dissipation = (mass of water crystal * (Q + heat of fusion)) / power where Q is the amount of heat required to increase the temperature of a substance (ice).

or

t(sec) = (m (kg) * Ht(kj/kg)) / P(watts)

where t is the time required for contrail dissipation(transformation), in seconds, m is the mass of the ice crystal in kilograms, Ht is the heat of transformation of ice in kilojoules per kilogram, and P is the power applied to the system in watts.

Calculating the internal energy, or enthalpy, of water vapor often involves several phase changes, as water varies between solid, liquid and vapor under varying conditions of temperature and pressure. In the case of a contrail composed of water vapor, the heat of transformation will consist of two phases. The first is the amount of heat required to raise the temperature of the ice crystal at a sub-zero temperature to 0 deg. C., which will be designated as Q in the present case. The second segment of heat required will be that which melts the ice crystal to a liquid form. The primary processes involved in contrail formation therefore appear to involve:

1. The emission of water vapor from the aircraft.
2. The freezing of the water vapor at sub-zero temperatures into ice crystals.
3. The warming of the ice crystals to the melting point through solar radiation.
4. The melting of the ice crystal with solar radiation to where the water vapor once again no longer is visible. This returns the water to the state from which it was emitted from the engine.

Let us now quantify the components of this model with elements that are typical or representative of the conditions of contrail formation:

Mass:

Assume that we have a cubed particle size (nucleated ice crystal) of dimension d on a side, measured in microns(designate as u). Given also that the density of ice is .917gm/cm3, the mass of the particle is:

mass=(d(u) * (1E-6m/u))^3 * (1E6cm3/m3) * (.917gm/cm3) * (1E-3kg/cm3)

or

mass = (d^3 * 9.17E-16 cm3 gm kg m3) / (m3 cm3 gm)

Q + Heat of Fusion:

Q is equal to the amount of heat required to increase the temperature of the ice crystal from the ambient temperature to 0 deg. C. The specific heat of ice is given as 4.21 kJ/(kg C) at 0 deg. C. The specific heat varies only slightly with respect to temperature and pressure, and this value will therefore be used. J refers to joules of energy.

The heat of fusion of ice is 335kJ/kg. It requires this amount of energy to melt ice.

Therefore, the amount of heat required to transform the ice crystal is:

dQ + heat of fusion = 4.21 kJ/(kg C) * dT + 355kJ/kg

where dQ is the amount of heat entering the ice crystal, the heat of fusion is the amount of heat required to melt the ice crystal, and dT is the temperature change from the ambient air to 0 deg. in Celsius.

The model now becomes:

t(sec) = (d^3 * (9.17E-16)cm^3 gm kg m^3 * ((((4.21kJ/kg)*dT)/(kg C)) + 355kj/kg)) / P * (m^3 cm^3 gm)

Power (P):

The energy of solar radiation is given in terms of watts/ square meter. Representative values measured range approximately from 200 to 700 watts/m^3. To arrive at the power applied to the ice crystal, we will take the surface area of the crystal exposed perpendicularly to the sunlight, and apply the solar radiation to it. The solar radiation will be applied on a continuous basis to the surface area until melting is complete.

Power absorbed = d^2 * (watts/m^2) * (1E-6m/u)^2

and since 1 watt = 1 joule/sec

Power absorbed = d^2 * (J/(m^2 s) * (1E-12) m^2/u^2

The model now becomes:

t(sec) = (d(u)^3 * (9.17E-16) cm3 gm kg m^3 * ((4.21kJ/kg * dT kJ/kg C) + (335kj/kg))) / (d(u)^2 * (J/(m^2 s) * (1E-12) m^2 / u^2)

Simplifying:

t(sec) = ((d(u) * (9.17E-13) * (4.21dT + 335) J cm^3 gm kg m^3 s m^2) / (Watts * 1E-12 J m^2 m^3 cm^3 gm kg)

or t(sec) = (d(u) * (9.17E-13) * (4.21dT + 335)) sec / (Watts * 1E-12)

or t(sec) = (d(u) * .917 * (-4.21T + 335)) / Watts/m2 where d is measured in microns, T is the air temperature where the contrail forms, measured in Celsius, and solar radiation is in watts per square meter.

Representative cases and the application of this model will now be considered. Research indicates that the expected size of particles emitted from aircraft ranges between 30 and 200 microns (Goethe MB – Ground Based Passive Remote Sensing of Ice Clouds with Scattered Solar Radiation in the Near Infrared – Max Planck Inst Meteorol). The temperature of the air at flight altitudes commonly approaches -50 deg. C. Solar radiation commonly ranges between 400 and 700 watts per square meter.

In the tables presented, d is the dimension of the ice crystal along one side of the cube, T is the temperature of the ambient air where the contrail forms (.e.g, 35000ft. MSL), and P is the solar radiation in Watts/sq. m. t is the length of time that it requires for the contrail, or ice crystal to dissipate (i.e., transform from ice to water vapor).

d(microns)	T(deg. C.)	P(watts/sq. m)	t(sec)
1	-50	600	1
10	-50	600	8
30	-50	600	25
50	-50	600	42
100	-50	600	83
1	-40	400	1
10	-40	400	12
30	-40	400	35
50	-40	400	58
100	-40	400	115
1	-30	700	1
10	-30	700	6
30	-30	700	18
50	-30	700	33
100	-30	700	60

This model covers the expected size range of any particles expected to be emitted by aircraft; most airborne particles range between 0-100 microns. It is of interest that the particle sizes considered in this model are generally considered to be too large to serve as cloud condensation nuclei; the average expected size of cloud condensation nuclei is extremely small, and on the order of .1 to .2 microns. A 10 micron particle is considered extremely large with respect to cloud condensation nuclei. This size distinction, when coupled with the results of the model above, further indicate the need to consider cloud formation as a separate and distinct physical process from that of contrail dissipation. That analysis would necessarily consider the significant role that aerosol particles, deliberately or otherwise introduced, would have on the cloud nucleation and formation process.

As can be seen, the results of this model agree extremely well with the observed properties of contrails over their historical existence. This work is based upon the physical processes, chemistry and mathematics of thermodynamics with respect to water and the various phase states. Consideration has also been given to the phenomenon of sublimation, and it has been found to be not applicable due to the extremely low atmospheric pressure requirements for sublimation to occur($P<.006$atm). The greatest variation within this model is seen to relate to particle size. It is seen that the contrails composed of the smaller particles dissipate within 30 seconds or less, and that the contrails composed of even relatively large particles are expected to dissipate within a couple of minutes at most.

If the dissipation of an observed contrail does not conform to the model above, and the corresponding physics and chemistry and math of same, then the logical conclusion that can be drawn is that the material of emission is not likely to be water vapor. As mentioned earlier, the physics of cloud formation are an entirely separate process, and are highly dependent upon temperature, relative humidity, aerosol type and the size of aerosol particles that are introduced. Any alterations in the formation of cloud processes as they have been repeatedly observed and recorded must necessarily consider the impact of these aerosols, identified and unidentified, within the analysis. Prior attention given to microscopic hydrated salts remains a priority in this research.

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September 16 2000
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